PREDICTION OF SOUND PRESSURE LEVELS ON ROCKET VEHICLES DURING ASCENT Revision B

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March 26, 2002

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## INTRODUCTION

Rocket vehicles are subjected to aerodynamic pressure excitation as they ascend in the atmosphere. The excitation sources include aerodynamic shockwaves, turbulent boundary layers, and recirculating flow. Furthermore, the excitation pressure fluctuates, with a broadband random characteristic.

Shockwaves begin to form along the rocket body as the vehicle approaches and surpasses the transonic velocity. Note that the local airflow must follow the geometry of the vehicle. Thus the local airflow reaches supersonic speed before the vehicle itself reaches supersonic speed.

The excitation is also severe as the vehicle encounters its maximum dynamic pressure condition.

The purpose of this report is to present empirical methods for determining the aerodynamic pressure power spectral density at stations along a rocket vehicle body for various flow regimes. The equations are based primarily on Reference 1.

The methods present are approximate. A particular problem is that shockwaves may traverse the length of the vehicle during the brief period that the vehicle accelerates through the transonic velocity.

The flow regimes are shown in Figures 1 through 9.

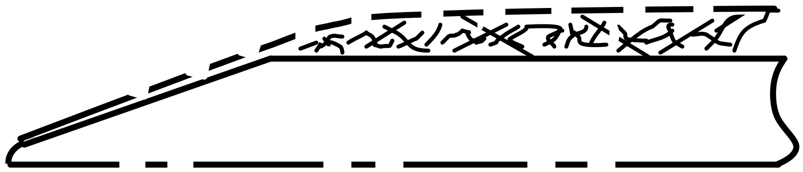

Figure 1.  Cone-Cylinder Geometry, Subsonic, Shoulder Separation

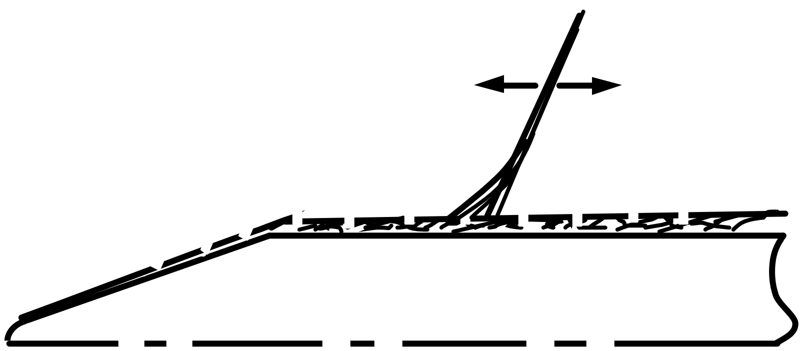

Figure 2.  Cone-Cylinder Geometry, Transonic Shockwave Oscillation with Attached Flow

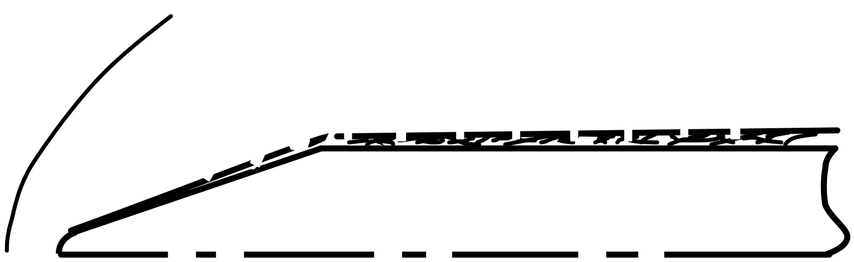

Figure 3.  Cone-Cylinder Geometry, Supersonic, Attached Flow

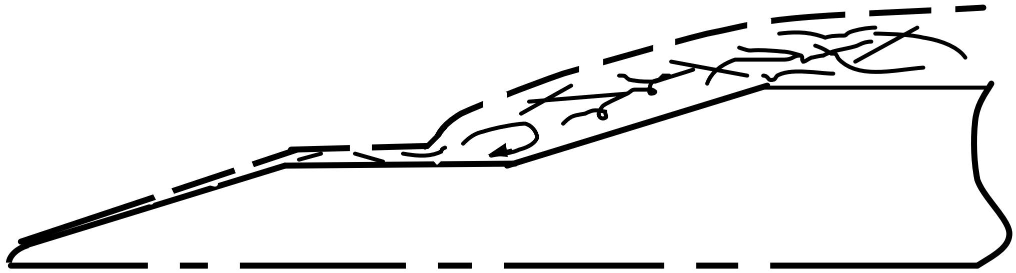

Figure 4.  Cone-Cylinder Geometry with Separated Flow near Compression Corner, Subsonic

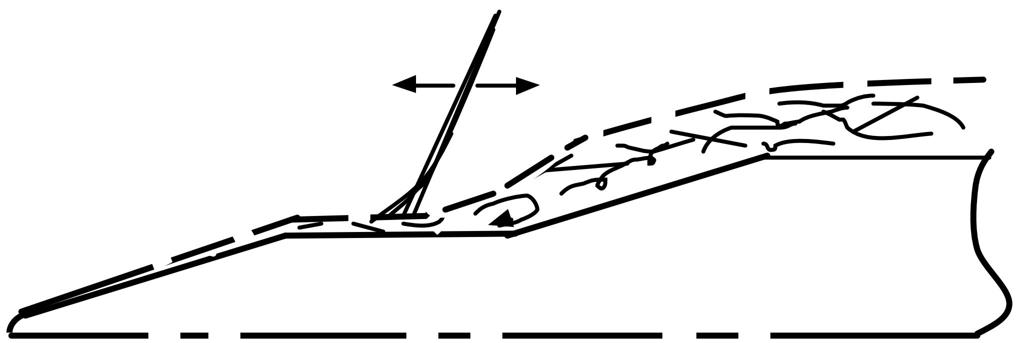

Figure 5. Cone-Cylinder Geometry with Separated Flow near Compression Corner, Transonic

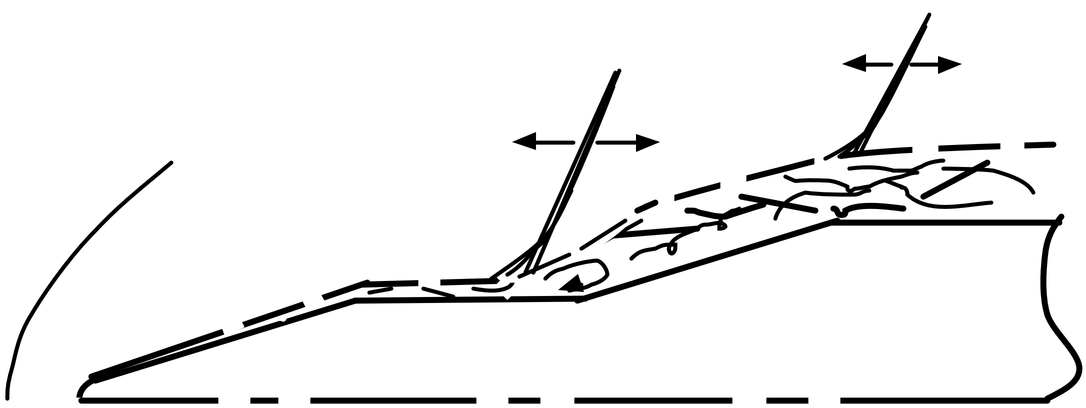

Figure 6. Cone-Cylinder Geometry with Separated Flow near Compression Corner and Shockwaves, Supersonic

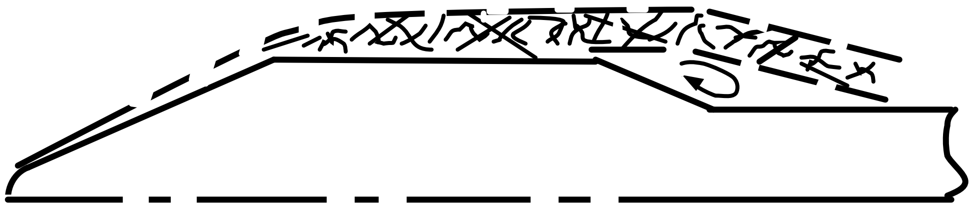

Figure 7. Shoulder and Boattail Induced Separation, Subsonic

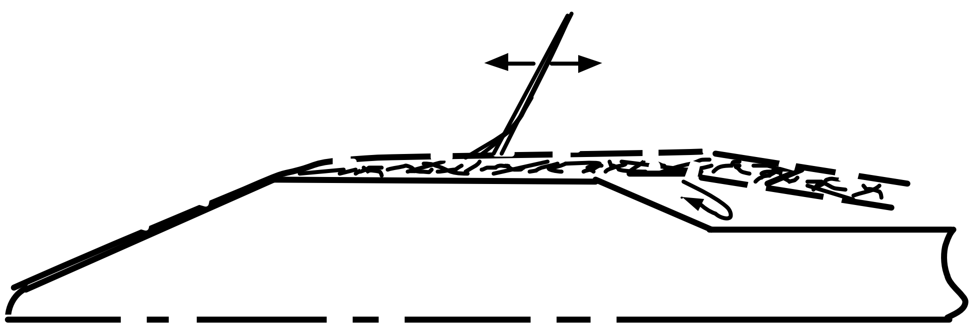

Figure 8. Shockwave Oscillation with Boattail Induced Separation, Transonic

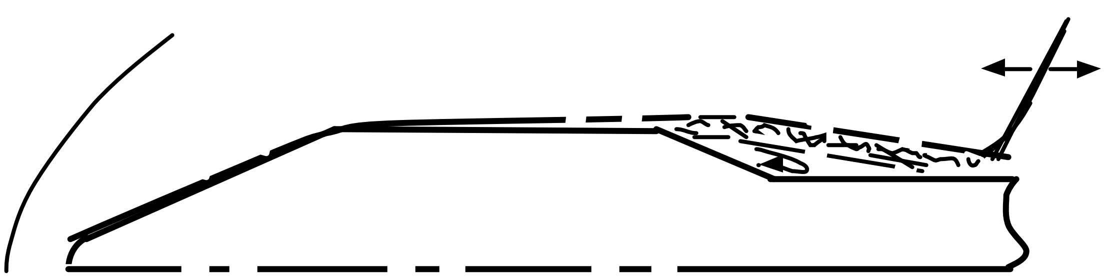

Figure 9. Attached Flow with Boattail Induced Separation and Shockwave Oscillation, Supersonic

ATTACHED FLOW

Boundary Layer Displacement Thickness

The boundary layer displacement thickness $\delta^*$ for an attached flow is

$$\frac{\delta^*}{x} = 0.0371 \left(R_{ex}\right)^{-0.2} \frac{\left(\frac{9}{7} + 0.475\, M^2\right)}{\left(1 + 0.13\, M^2\right)^{0.64}} \qquad (1)$$

where

M = Mach number

$R_{ex}$ = Reynolds number based on distance x

x = distance from the start of boundary layer growth

Equation (1) is taken from References 1 and 2.

The boundary layer thickness $\delta$ is the thickness for which $u = 0.99\ u_\infty$.

The displacement thickness $\delta^*$ is the thickness in a frictionless flow that would yield the same mass flow rate in a viscous flow.

Note that the Reynolds number for flow over a flat plate is

$$R_{ex} = \frac{U_\infty\, x}{\nu} \qquad (2)$$

where

$U_\infty$ = free stream velocity

x = distance from the start of boundary layer growth

$\nu$ = kinematic viscosity

Equation (2) taken from Reference 3.

RMS Pressure for Attached Flow

The RMS pressure $p_{rms}$ for an attached boundary layer is given by

$$\frac{p_{rms}}{q} = \frac{0.010}{F} \qquad (3)$$

where

$q$ = dynamic pressure

$F$ $= 0.5 + \left(\frac{T_w}{T_{aw}}\right)\left(0.5 + 0.09M^2\right) + 0.04M^2$

$T_w$ = wall temperature

$T_{aw}$ = adiabatic wall temperature

The ratio $T_w / T_{aw}$ is often taken as unity as a simplifying assumption.

The dynamic pressure q is

$$q = \frac{1}{2} \rho U_\infty^2 \qquad (4)$$

where $\rho$ is the density of the gas or fluid.

Note that the Mach number is related to the free stream velocity by

$$M = \frac{U_\infty}{c} \qquad (5)$$

where c is the speed of sound.

The speed of sound varies with temperature, and hence with altitude, as shown in Appendix A.

Power Spectral Density for Attached Flow

The power spectral density G(f) for attached flow is

$$G(f)\,\frac{U_\infty}{q^2\,\delta^*} = 4\left[\frac{p_{rms}}{q}\right]^2\left[\frac{F^{1.433}}{1+F^{2.867}\,\Omega^2}\right] \tag{6}$$

where

f = frequency

$$\Omega = \frac{2\pi f\,\delta^*}{U_\infty}$$

Equation (6) is taken from References 1 and 4.

## SEPARATED FLOW AND SHOCKWAVES

Compensation Factor

A compensation factor $C_{comp}$ is needed to account for a low frequency shift of energy for separated flow and shockwaves.

Compression Corner Plateau Region, Transonic Flow

The compensation factor is

$$C_{comp} = 3 \tag{7}$$

The compensated RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{comp} = \frac{0.025}{\left[1+M^2\right]} \tag{8}$$

Equation (8) is taken from Reference 1.

Compression Corner Reattachment Region, Transonic Flow

The compensation factor is

$$C_{comp} = 9 \tag{9}$$

The compensated RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{comp} = \frac{0.10}{\left[1 + M^2\right]} \tag{10}$$

Equation (10) is taken from Reference 1. Note that the ratio is four times greater for the reattachment region than for the plateau region, for transonic flow at a compression corner.

Compression Corner Plateau Region, Supersonic Flow

The compensation factor is

$$C_{comp} = 10 \tag{11}$$

The pressure ratio for the shockwave at the separation point must be considered for this case.

Let

$P_1$ = static pressure upstream of shockwave

$P_2$ = static pressure downstream of shockwave

$\alpha$ = frustum angle

$M_1$ = upstream Mach number

Define an angle $\theta$ as

$$\theta = \alpha + \arcsin\left[\frac{1}{M_1}\right] \tag{12}$$

The pressure ratio is

$$\left[\frac{P_2}{P_1}\right] = \frac{1}{2.4}\left[2.8 M_1{}^2 \sin^2\theta \;\; - \; 0.4\right] \tag{13}$$

Equation (13) is taken from Reference 1.

The turbulent boundary layer RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{tbl} \;\; = \; \frac{0.006}{F} \tag{14}$$

Equation (14) is taken from References 1 and 4.

The compensated RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{comp} \;\; = \; \left[\frac{p_{rms}}{q}\right]_{tbl} \left[\frac{P_2}{P_1}\right] \tag{15}$$

Compression Corner Separation or Reattachment Shockwave

The compensation factor is

$$C_{comp} = 30 \tag{16}$$

The pressure ratio is

$$\left[\frac{P_2}{P_1}\right] = \frac{1}{2.4}\left[2.8 M_1{}^2 \sin^2\theta \;\; - \; 0.4\right] \tag{17}$$

The turbulent boundary layer RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{tbl} \;\; = \; \frac{0.006}{F} \tag{18}$$

The RMS shock pressure is

$$\left[\frac{(p_{rms})_{shock}}{(p_{rms})_{tbl}}\right] = -1.181 + 1.713\left[\frac{P_2}{P_1}\right] + 0468\left[\frac{P_2}{P_1}\right]^2 \qquad (19)$$

Equation (19) is taken from References 1 and 4.

$$\left[\frac{p_{rms}}{q}\right]_{shock} = \left[\frac{p_{rms}}{q}\right]_{tbl}\left[\frac{(p_{rms})_{shock}}{(p_{rms})_{tbl}}\right] \qquad (20)$$

The compensated RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{comp} = \left[\frac{p_{rms}}{q}\right]_{tbl}\left[\frac{(p_{rms})_{shock}}{(p_{rms})_{tbl}}\right] \qquad (21)$$

<u>Power Spectral Density for Compression Corner</u>

The power spectral density G(f) for a compression corner is

$$G(f)\,\frac{U_\infty}{q^2\,\delta^*} = 4\left[\left(\frac{p_{rms}}{q}\right)_{comp}\right]^2 C_{comp}\left[\frac{F^{1.433}}{1+(C_{comp})^2\,F^{2.867}\,\Omega^2}\right] \qquad (22)$$

where

f = frequency

$$\Omega = \frac{2\pi f\,\delta^*}{U_\infty}$$

Equation (22) is taken from Reference 1.

EXPANSION CORNER

Compensation Factor

A compensation factor $C_{exp}$ is needed to account for a low frequency shift of energy for expansion corner flow.

Expansion Corner Plateau Region, Transonic and Supersonic Flow

The compensation factor for an expansion corner plateau region is

$$C_{exp} = 3 \tag{23}$$

The RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{exp} = \frac{0.040}{\left[1+M^2\right]} \tag{24}$$

Equation (24) is taken from References 1 and 5.

Expansion Corner Reattachment Region, Transonic Flow

The compensation factor for an expansion corner reattachment region is

$$C_{exp} = 9 \tag{25}$$

The RMS pressure is

$$\left[\frac{p_{rms}}{q}\right]_{exp} = \frac{0.16}{\left[1+M^2\right]} \tag{26}$$

Equation (26) is taken from Reference 1.

Power Spectral Density for Expansion Corner

The power spectral density G(f) for an expansion corner is

$$G(f)\ \frac{U_\infty}{q^2\ \delta^*}\ =\ 4\left[\left(\frac{p_{rms}}{q}\right)_{exp}\right]^2 C_{exp}\left[\frac{F^{1.433}}{1+\left(C_{exp}\right)^2 F^{2.867}\ \Omega^2}\right] \tag{27}$$

Equation (27) is taken from Reference 1.

REFERENCES


1. Wilby, J. and Wilby E., Prediction of External Fluctuating Pressure Field on Taurus During Ascent, AARC Report No. 131, Van Nuys, California, 1991
2. Laganelli, A.L., Howe, J.R., "Prediction of Pressure Fluctuations Associated with Maneuvering Re-Entry Weapons," AFFDL-TR-77-59, Vol I, July 1977.
3. Incropera and DeWitt, Fundamentals of Heat Transfer, Wiley, New York, 1981.
4. Laganelli, A.L., Wolfe, H.F., "Prediction of Fluctuating Pressure in Attached and Separated Turbulent Boundary Layer Flow," AIAA Paper AIAA-89-1064, April 1989.
5. Robertson, J.E., "Prediction of In-Flight Fluctuation Pressure Environments Including Protuberance Induced Flow," Wyle Laboratories Report WR 71-10, March 1971.

# APPENDIX A

## Variation of the Speed of Sound in the Atmosphere with Altitude

The pressure, temperature, density and speed of sound for the international standard atmosphere (ISA) can be calculated for a range of altitudes from sea level upward. These parameters are obtained from the hydrostatic equation for a column of air. The air is assumed to be a perfect gas.

The atmosphere consists of two regions.

The troposphere is the region between sea level and an altitude of approximately 11 km (36,089 feet). In reality, the boundary may be at 10 to 15 km depending on latitude and time of year. The temperature lapse rate in the troposphere is taken as L= 6.5 Kelvin/km. The actual value depends on the season, weather conditions, and other variables.

The stratosphere is the region above 11 km and below 50 km. The stratosphere is divided into two parts for the purpose of this tutorial.

The lower stratosphere extends from 11 km to 20 km. The temperature remains constant at 217 Kelvin (-69.1 F) in the lower stratosphere.

The upper stratosphere extends from 20 km to 50 km. The temperature rises in the upper stratosphere.

## Basic Equations

The hydrostatic equation for pressure P and altitude h is

$$\frac{dP}{dh} = -\rho g \tag{A-1}$$

where

$\rho$ = mass density,
g = gravitational acceleration.

The perfect gas equation is

$$P = \rho \frac{R}{M} T_k \tag{A-2}$$

where

R is the universal gas constant,
M is the molecular weight,
$T_k$ is the absolute temperature in Kelvin.

Note that for air,

$$\frac{R}{M} = \frac{8314.3 \text{ J/(kgmole} \cdot \text{K)}}{28.97 \text{ kg/kgmole}} \tag{A-3}$$

$$\frac{R}{M} = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}} \tag{A-4}$$

$$\frac{R}{M} = 287 \frac{\text{m}^2/\text{sec}^2}{\text{K}} \tag{A-5}$$

<u>Troposphere</u>

The temperature lapse equation for the troposphere is

$$T = T_o - Lh \tag{A-6}$$

Recall the formula for the speed of sound in a perfect gas.

$$c = \sqrt{\gamma\left(\frac{R}{M}\right)T_k} \tag{A-7}$$

The speed of sound in the troposphere is thus

$$c = \sqrt{\gamma\left(\frac{R}{M}\right)(T_o - Lh)} \tag{A-8}$$

The standard sea level temperature is $T_o = 288$ Kelvin. Again, L= 6.5 Kelvin / km for the troposphere.

Substitute equation (A-6) into (A-2) to obtain the perfect gas law for the troposphere.

$$P = \rho \frac{R}{M}[T_o - Lh] \tag{A-9}$$

The density in the troposphere can thus be expressed as

$$\rho = \frac{P}{\frac{R}{M}\left[T_o - Lh\right]} \tag{A-10}$$

Solve the hydrostatic equation for a constant lapse rate. The resulting equation gives the pressure variation with altitude. Neglect the variation of gravity with altitude. Rewrite equation (A-1).

$$dP = -\rho g\, dh \tag{A-11}$$

Substitute equation (A-10) into (A-11).

$$dP = -\frac{P}{\frac{R}{M}\left[T_o - Lh\right]} g\, dh \tag{A-12}$$

$$\frac{dP}{P} = -\frac{g}{\frac{R}{M}\left[T_o - Lh\right]}\, dh \tag{A-13}$$

The hat sign is added in order to prevent confusion between the integration variables and the limits.

$$\int_{P_o}^{P} \frac{d\hat{P}}{\hat{P}} = -\int_{0}^{h} \frac{g}{\frac{R}{M}\left[T_o - L\hat{h}\right]}\, d\hat{h} \tag{A-14}$$

$$\ln\left[\hat{P}\right]\Big|_{P_o}^{P} = \frac{Mg}{LR} \ln\left[T_o - L\hat{h}\right]\Big|_{0}^{h} \tag{A-15}$$

$$\ln\left[\frac{P}{P_o}\right] = \frac{Mg}{LR}\left\{\ln\left[T_o - Lh\right] - \ln\left[T_o\right]\right\} \tag{A-16}$$

$$\ln\left[\frac{P}{P_o}\right] = \frac{Mg}{LR}\left\{\ln\left[\frac{T_o - Lh}{T_o}\right]\right\} \tag{A-17}$$

$$\left[\frac{P}{P_o}\right] = \left[\frac{T_o - Lh}{T_o}\right]^{\left[\frac{Mg}{LR}\right]} \tag{A-18}$$

The pressure in the troposphere is thus

$$P = P_o\left[\frac{T_o - Lh}{T_o}\right]^{\left[\frac{Mg}{LR}\right]} \tag{A-19}$$

Note that the sea level pressure is $P_o = 101.3\text{kPa}$.

The density in the troposphere is obtained from equations (A-10) and (A-19).

$$\rho = \frac{P_o\left[\frac{T_o - Lh}{T_o}\right]^{\left[\frac{Mg}{LR}\right]}}{\frac{R}{M}\left[T_o - Lh\right]} \tag{A-20}$$

<u>Lower Stratosphere</u>

Again, the temperature is constant in the lower stratosphere. The speed of sound is thus constant in the lower stratosphere.

$$dP = -\rho g\, dh \tag{A-21}$$

Let $T_c$ be the constant temperature in the lower stratosphere.

$$dP = -\frac{P}{\frac{RT_c}{M}} g\, dh \tag{A-22}$$

$$\frac{dP}{P} = -\frac{Mg}{RT_c}\, dh \tag{A-23}$$

The hat sign is added in order to prevent confusion between the integration variables and the limits.

$$\int_{P_1}^{P} \frac{d\hat{P}}{\hat{P}} = -\int_{h_1}^{h} \frac{Mg}{RT_c} d\hat{h} \tag{A-24}$$

$$\ln\left[\hat{P}\right]\Big|_{P_1}^{P} = \frac{-Mg}{RT_c}\hat{h}\Big|_{h_1}^{h} \tag{A-25}$$

$$\ln\left[\frac{P}{P_1}\right] = \frac{-Mg}{RT_c}\left[h - h_1\right] \tag{A-26}$$

$$\frac{P}{P_1} = \exp\left\{\frac{-Mg}{RT_c}\left[h - h_1\right]\right\} \tag{A-27}$$

The pressure in the lower stratosphere is thus

$$P = P_1 \exp\left\{-\frac{Mg}{RT_c}\left[h - h_1\right]\right\} \quad \text{(A-27)}$$

Note that $P_1$ is the pressure at the lower altitude limit of the stratosphere.

The density in the lower stratosphere is thus

$$\rho = \frac{M}{RT_k} P_1 \exp\left\{-\frac{Mg}{RT_c}\left[h - h_1\right]\right\} \tag{A-28}$$

<u>Summary</u>

The pressure, density, and speed of sound are given in Table A-1 for an altitude up to 20 km.

| Table A-1. Atmospheric Properties | | | | | |
|---|---|---|---|---|---|
| Altitude (km) | Pressure (kPa) | Mass Density ($kg/m^3$) | Temp. (Kelvin) | Temp. (°C) | Speed of Sound (m/sec) |
| 0 | 101.3 | 1.226 | 288 | 14.9 | 340.2 |
| 1 | 89.85 | 1.112 | 282 | 8.4 | 336.3 |
| 2 | 79.47 | 1.007 | 275 | 1.9 | 332.4 |
| 3 | 70.09 | 0.9096 | 269 | -4.7 | 328.5 |
| 4 | 61.62 | 0.8195 | 262 | -11.2 | 324.5 |
| 5 | 54.00 | 0.7365 | 256 | -17.7 | 320.4 |
| 6 | 47.17 | 0.6600 | 249 | -24.2 | 316.3 |
| 7 | 41.05 | 0.5898 | 243 | -30.7 | 312.1 |
| 8 | 35.59 | 0.5254 | 236 | -37.2 | 307.9 |
| 9 | 30.73 | 0.4666 | 230 | -43.7 | 303.7 |
| 10 | 26.43 | 0.4129 | 223 | -50.2 | 299.3 |
| 11 | 22.62 | 0.3641 | 217 | -56.2 | 295 |
| 12 | 19.33 | 0.3104 | 217 | -56.2 | 295 |
| 13 | 16.51 | 0.2652 | 217 | -56.2 | 295 |
| 14 | 14.11 | 0.2266 | 217 | -56.2 | 295 |
| 15 | 12.06 | 0.1936 | 217 | -56.2 | 295 |
| 16 | 10.30 | 0.1654 | 217 | -56.2 | 295 |
| 17 | 8.801 | 0.1413 | 217 | -56.2 | 295 |
| 18 | 7.519 | 0.1207 | 217 | -56.2 | 295 |
| 19 | 6.424 | 0.1032 | 217 | -56.2 | 295 |
| 20 | 5.489 | 0.0881 | 217 | -56.2 | 295 |

Again, the values in Table A-1 are approximate. The actual values depend on the time of day, season, weather conditions, etc.