On Mac: `CCD's/qe_res_may_jun_05/res_revisit_08dec05`

Point spread function in a depleted CCD revisited

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The mobility μ is defined as the proportionality constant relating drift velocity v_d to the electric field E:

$$v_d = \mu E \tag{1}$$

It is strongly temperature dependent, For high purity silicon at low fields, Jacobini *et al.*[1](and, presumably, his reference Conwell[2]) give

$$\mu\,(T, E \approx 0) = A\,T^{-\gamma}\ , \tag{2}$$

where for holes $A = 1.35\,\mathrm{cm}^2\mathrm{K}^\gamma\mathrm{V}^{-1}\mathrm{s}^{-1}$ and $\gamma = 2.20$. But v_d is not a linear function of E. We write

$$\mu\,(T, E) \equiv \mu\,(T, 0)\ m\,(T, E)\ \ . \tag{3}$$

Canali *et al.*[3] obtain a good fit to experimental data with

$$m\,(T, E) \approx \left[1 + (E/E_c)^\beta\right]^{-1/\beta}\ \ , \tag{4}$$

where, again for holes, $E_c = 1.24\,T^{1.68}$, and $\beta = 0.46\,T^{\,0.17}$. As can be seen, E_c is the saturation field at which v_d becomes constant, For the case of the LBNL CCD's, E is typically $(100\,\mathrm{V})/(200\,\mu\mathrm{m})$ $= 5\,\mathrm{kV/cm}$; the nonlinearity is important,

The diffusion constant D is also temperature and field dependent; we define the field correction $\delta(T, E)$ by

$$D\,(T, E) = D\,(T, 0)\ \delta\,(T, E)\ \ . \tag{5}$$

Jacobini *et al.* indicate in their Fig. 13, for holes at 300 K, D decreases by a factor of 0.5_5 between 0 and 5 kV/cm. At that time experimental data were not available for other temperatures, and we have not yet found the relevant data. (Transverse diffusion of holes.) This decrease would decrease the variance by a factor of 0.7_5, nearly canceling the increase via $m(T, E)$. Pending better information, we assume that the correction factor $\delta(T, E)$ is unity.

In thermal equilibrium the Einstein equation relates D and μ: $D/\mu = kT/q$. In the high-field case with "hot" carriers, this equilibrium does not obtain, so we must use $D(T, 0)/\mu(T, 0) = kT/q$.

We are interested in the transverse diffusion of holes produced near the back of an overdepleted CCD (thickness y_D) with electric field $E(y)$ given by Eqns. 4 and 5 in Ref. 4. The variance σ increases by

$$\begin{aligned} d\sigma^2 &= 2D\,dt \\ &= 2\,D\frac{dy}{v_d} \\ &= \frac{2kT}{q}\,\frac{\delta\,(T, E)}{m\,(T, E)\ E\,(y)}dy \end{aligned} \tag{6}$$

in the time dt that elapses while the hole moves from y to $y + dy$. The variance is thus

$$\sigma^2 = \frac{2kT}{q}\int_0^{y_D} \frac{\delta\,(T, E)}{m\,(T, E)\ E\,(y)}dy\ . \tag{7}$$

In the asymptotic case

$$E(y) \to \langle E(y) \rangle = V_{\rm appl}/y_D = (V_{\rm sub} - V_{J'})/y_D ,$$

and

$$\sigma^2_{\rm asymp} = \frac{2kT}{q} \frac{\delta(\langle E \rangle, T)\, y_D^2}{m(\langle E \rangle, T)\,(V_{\rm sub} - V_{J'})} . \tag{8}$$

(to be extended after we learn something about $\delta(T, E)$.

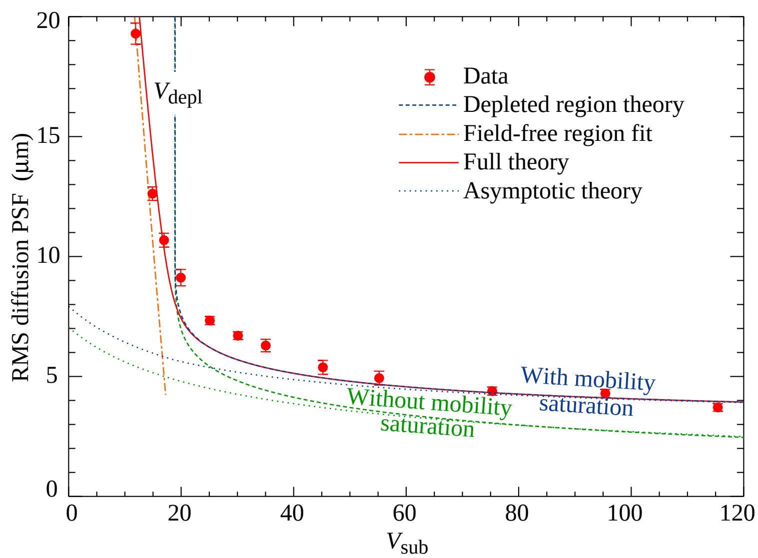

Figure 1: Effect of mobility saturation from Eq. (7). $m(T, E) = 1$ for the case marked "Without mobility saturation," and with Canali's form (Eq. (4)) for the case marked "With mobility saturation." There are no parameters in the theoretical curve except for $V_{J'}$, to which the higher-voltage result is insensitive. In both cases, the diffusion is field-independent, *i.e.*, $\delta(T, E) = 1$. The experimental data were reported in Ref. 5. The discrepant results for 25–50 V are not understood.

References:

1. C. Jacobini *et al.*, "A review of some charge transport properties of silicon," Solid-State Electronics **20**, 77–89 (1977).
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3. C. Canali, G. Majni, R. Minder & G. Ottaviani, IEEE Trans. Electron Dev. **ED-22**, 1045 (1975).
4. S.E. Holland, D.E. Groom, N.P. Palaio, R. J. Stover, and M. Wei, IEEE Trans. Electron Dev. 50 (3), 225–238 (January 2003).
5. J. A. Fairfield, *et al.*, "Improved spatial resolution in thick, fully depleted CCDs with enhanced red sensitivity," IEEE Trans. Nucl. Sci. 53 (6), 3877-3881 (2006).